Origin of "Erosional Velocity Formula"

The erosional velocity cited as $V_{max} = \frac{c}{\sqrt{\rho}}$ in different companies' documents and also API RP14E. This velocity is one of restrictions for high velocity in pipes. Different numbers are mentioned for C factor depends on the situation. However, one important issue which should be considered is the unit of "c". In Imperial system it is $lb_m^{0.5}\ ft^{-0.5}\ s^{-1}$ and in SI system it is $Kg^{0.5}\ m^{-0.5}\ s^{-1}$. Numbers in SI unit is about 1.22 times bigger than Imperial's.

When stream flows it exert a shear on the internal wall of pipe. This shear removes small chips from the wall, make it thinner and decrease the lifetime the pipe. This shear is higher if the fluid denser (lower density) and or fluid carries some eroding material like sand. From the other side the softer the pipe material the more susceptible for erosion.

Based on the above discussion, the "c" factor is a function of the below parameters:

-Density of the fluid
-Erosive content of the fluid
-Corrosivity of the fluid (it increases the erosional corrosivity)
-Material of pipe
-Wall thickness (which includes corrosion allowance)
-Pipe frequency of usage (e.g. continuous vs. intermittent)

Proof:
In a circular pipe shear causes the pressure loss. The shear on the pipe is related to pressure loss of the pipe as per below equation:

$$\Delta P = 4\frac{L}{D}.\tau_w$$

Where τ_w is shear on wall pipe
From the other side the pressure loss can be calculated by:

$$\Delta P = f\frac{L}{D}.\frac{\rho v^2}{2}$$

After substitution:

$$f.\frac{\rho v^2}{2} = 4\tau_w$$

Or:

$$v = \frac{\sqrt{8\tau_w}}{\sqrt{f}.\sqrt{\rho}}$$

In high velocity cases the flow is fully turbulent and Moody friction factor, f, is independent of Reinhold number and only the function of e/d or roughness to diameter ratio. A conservative f happens in the minimum e/d. For a minimum practical pipe diameter of 2" and roughness of 0.0018in for carbon steel, e/d will be 0.001.

F factor is 0.002 for e/d=0.001 based on Moody diagram.

Assume V_{max} when τ_w = 40 N/m^2

$$v_{max} = \frac{\sqrt{8*40}}{\sqrt{0.02}.\sqrt{\rho}}$$

$$V_{max} = \frac{126}{\sqrt{\rho\ \ kg/m^3}}$$

Or in general:

$$V_{max} = \frac{c}{\sqrt{\rho}}$$

C				
SI Units	Imperial Units	Fluid	Pipe Material	Usage
122	100	Clean fluid	Carbon steel	Continuous
152	125	Clean fluid	Carbon steel	Intermittent
244	200	Clean fluid	Corrosion resistant material	Continuous